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Articles

Modeling of bumping routes in the RSK algorithm and analysis of their approach to limit shapes

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Introduction: *The RSK algorithm establishes an equivalence of finite sequences of elements of linearly ordered sets and pairs of Young tableaux P and Q of the same shape. Of particular interest is the study of the asymptotic limit, i. e., the limit shape of the so-called bumping routes formed by the boxes of tableau P affected in a single iteration of the RSK algorithm. The exact formulae for these limit shapes were obtained by D. Romik and P. Śniady in 2016. However, the problem of investigating the dynamics of the approach of bumping routes to their limit shapes remains insufficiently studied.* ***Purpose:*** *To study the dynamics of distances between the bumping routes and their limit shapes in Young tableaux with the help of computer experiments.* ***Results:*** *We have obtained a large number of experimental bumping routes through a series of computer experiments for Young tableaux P of sizes up to* $4\cdot10^6$*, filled with real numbers in the range [0, 1] and sets of inserted values* $\alpha \in [0.1, 0.15, \ldots, 0.85]$*. We have compared these bumping routes in the* L_2 *metric with the corresponding limit shapes and have calculated the average distances and variances of their deviations from the limit shapes. We refined the parameters in the empirical formula for the rate of approach of discretized bumping routes to their limit shapes. Also, the experimental parameters of the normal distributions of the deviations of the bumping routes are obtained for various input values.*

Keywords *— RSK algorithm, RSK correspondence, Young tableau, Plancherel measure, Vershik — Kerov curve, bumping route, limit shape, linearly ordered set, numerical experiment, asymptotic combinatorics.*

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Introduction

The Robinson — Schensted — Knuth (RSK) algorithm [1–3] appears in various contexts of mathematics, physics, computer science and their applications [4, 5]. Despite its purely combinatorial nature, RSK has many deep connections with algebra, ergodic theory, and the theory of dynamical systems. In these days, more and more papers appear in which new and unexpected properties of this algorithm are revealed.

The geometry and combinatorics of the bumping routes of the RSK algorithm was studied [6–10] both theoretically and with the help of computer experiments. In [6], the asymptotic behaviour of the convergence of bumping routes to their limit shapes was studied, and exact equations describing such limit shapes were also obtained. In this paper, we study the dynamics of convergence of the generated bumping routes to their limit shapes. Similar simulations have already been performed in [9], but compared to this work, the scheme of experiments was significantly changed. In particular, the number of Young tableaux involved in the simulation was increased, and a larger number of limit shapes of bumping routes, to which the experimental curves tend, was considered. Here we chose another method for comparing the experimentally obtained bumping routes with their corresponding limit curves. Since the end of the limit shape of a bumping route may not coincide with the end of the experimentally calculated bumping route, the projections of these functions onto the x axis are generally defined on different intervals. At the same time, in [9] to calculate the deviation of a bumping route from the theoretical limit shape, we scaled the experimental bumping route by multiplying its domain of definition by some constant so that its end coincides with the theoretical one. Finally, the L_2 distance between the scaled experimental bumping route and the theoretical curve was calculated.

This approach led to a significant divergence of the projections of the curves on the y axis and, as a result, to an inaccurate estimation of the distance between the curves. Therefore, we subsequently abandoned such a scheme. In this article, these curves are compared on the common interval of their definition, i. e., the maximum common value of their projections on the x axis is chosen. Then the distance between the theoretical and experimental curves in the L_2 metric is evaluated. This distance is calculated at some fixed number of points on this common definition interval.

RSK algorithm

The classical RSK algorithm and its generalizations [1–3] consist of mapping some input sequence of elements of some linearly ordered set to a pair of Young tableaux of the same shape: insertion tableau P and recording tableau Q. The entries of the input sequence are processed one by one. If the next entry is the maximum in the first column of the tableau P, then a new box is added to the tableau on top of the first column. Otherwise, the element is written to the box of the first column with the closest greater value, which is then bumped into the second column. Likewise, the bumped value can either take the position at the top of the second column, or bump the closest greater value to the third one. The iteration of the algorithm ends when the next bumped value turns out to be the maximum in column j and a box with coordinates (i, j) is added to the tableau P. After that, a box filled by the index of the current element of the input sequence adds to tableau Q at the same position (i, j). Thus, the insertion tableau consists of inserted values and is *semi-standard Young tableau* (SSYT) [11, 12], i. e., its values strictly increase along the columns and non-strictly along the rows. The recording tableau is a *standard Young tableau* (SYT), i. e. the values in it are strictly ordered both in rows and columns.

Figure 1 shows an example of the RSK correspondence between a finite sequence of integers ω and a pair of Young tableaux P, Q. The arrows connect the boxes through which the values are bumped during one iteration of the RSK algorithm when processing the number 9. The sequence of such boxes is called the *bumping route*.

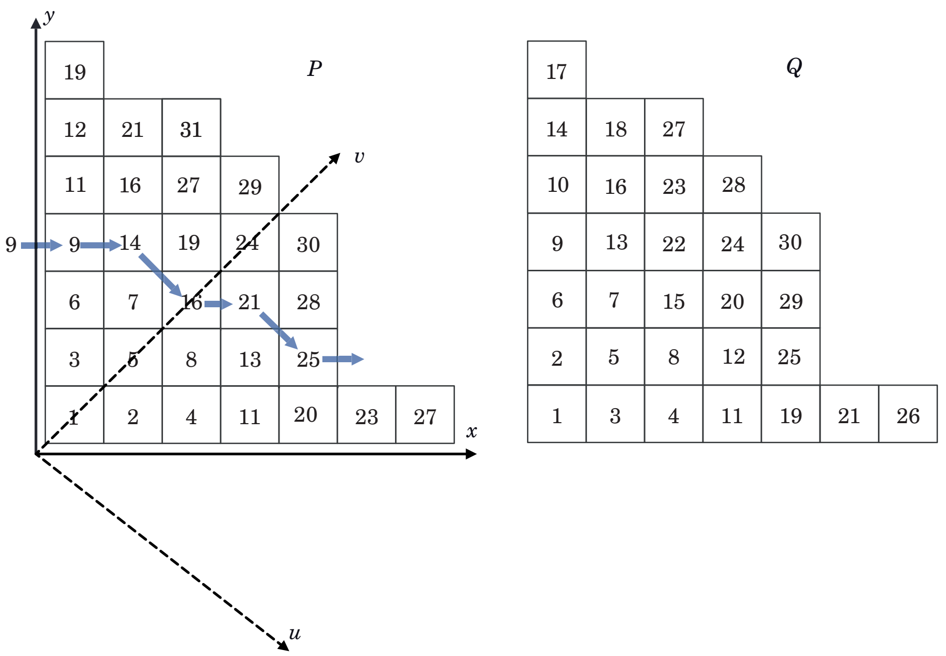
In this work, we use the so-called *French notation* for Young tableaux such as the boxes are aligned along the x and y axes. Another notation often used in literature is known as *Russian notation*. It is obtained from the French one by rotating the coordinate axes by 45 degrees clockwise. In Fig. 1, French notation corresponds to the x, y axes, and Russian notation corresponds to the u, v axes.

One of important properties of the RSK algorithm is the fact that when applied to a sequence of random uniformly distributed numbers, the resulting tableaux P and Q have the so-called *Plancherel distribution* [13]. When the size of the tableaux grows, their profiles tend to some limit shape Ω, which is described by the Vershik — Kerov curve [14], given in Russian notation as follows:

$$\Omega(u) = \frac{2}{\pi}\left(u \cdot \arcsin\frac{u}{2} + \sqrt{4-u^2} \right). \quad (1)$$

The limit shapes of the bumping routes are determined by the following formulae [6]:

ω = 11, 23, 4, 2, 5, 27, 25, 8, 20, 28, 1, 13, 21, 30, 16, 24, 31, 29, 3, 9, 7, 14, 19, 16, 11, 6, 27, 21, 12, 19, 9



■ ***Fig. 1.*** RSK correspondence between an integer sequence and a pair of Young tableaux

$$F(u)=\frac{1}{2}+\frac{1}{\pi}\left(\frac{u\sqrt{4-u^2}}{4}+\arcsin\frac{u}{2}\right),\ |u|\le 2; \quad (2)$$

$$u_\alpha(t)=\sqrt{t}\cdot F^{-1}\left(\frac{\alpha}{t}\right),\ 0\le\alpha\le t\le 1; \quad (3)$$

$$v_\alpha(t)=\sqrt{t}\cdot\Omega\left(F^{-1}\left(\frac{\alpha}{t}\right)\right),\ 0\le\alpha\le t\le 1; \quad (4)$$

$$y_\alpha(t)=\frac{v_\alpha(t)+u_\alpha(t)}{2}; \quad (5)$$

$$x_\alpha(t)=\frac{v_\alpha(t)-u_\alpha(t)}{2}; \quad (6)$$

$$\kappa(\alpha)=x_\alpha(1)=\frac{\Omega(F^{-1}(\alpha))-F^{-1}(\alpha)}{2}; \quad (7)$$

$$\beta_\alpha(s)=y_\alpha(x_\alpha^{-1}(s)),\ 0\le s\le\kappa(\alpha), \quad (8)$$

where α is the input value; t is the intermediate value on the bumping route; $u_\alpha(t)$, $v_\alpha(t)$, $x_\alpha(t)$, $y_\alpha(t)$ are the projections of the coordinates of the position where the value t was bumped when the input value α was processed onto the corresponding axes of the Russian and French notation; $\kappa(\alpha)$ is the projection of the intersection point of the bumping route and the Vershik — Kerov curve (1) onto the x axis; $\beta_\alpha(s)$ is the projection of the position on the bumping route onto the y axis.

The goal of this article is to study the deviations of the experimental bumping routes from their limit shapes. We consider sequences consisting of random real numbers uniformly distributed in the interval [0, 1].

Computer experiments

Let us describe the computational experiments on modeling the bumping routes in the RSK algorithm and studying their deviations from the limit shapes. With the help of the RSK algorithm, we generated a set of random pairs of SSYT P and SYT Q of some fixed size n, distributed according to the Plancherel measure [13]. Then, again using the RSK algorithm, a certain value of α was inserted into each of the tableaux P. The bumping route corresponding to this value was calculated, i. e., the sequence of normalized coordinates of the boxes $(x_i,\ y_i)$, bumped during the iteration of the algorithm. We divide the coordinates by $\sqrt{n}$ to normalize the area under the profile of the tableau to make it equal to one. Such a normalization is necessary to compare these bumping routes with the limit curves, which are defined by formulae (1)–(8).

As can be seen from formulae (3), (4), in order to calculate a coordinate on the limit shape of a bumping route, it is necessary to calculate the function inverse to F from (2). To solve this problem, we formed a table T_1, in which each value $u \in [-2, \ldots, 2]$, listed with the step $4\cdot 10^{-6}$, was associated with the corresponding value of the function F. To calculate $u = F^{-1}$, the row with the closest value to F was selected in T_1. Similarly, for the calculation $x_\alpha^{-1}(s)$ from (8), a series of tables T_α were compiled, for the values of α defined in the range $[\alpha, \ldots, 1]$ with the step 10^{-6}. So, each of T_α specifies the correspondence $t(x_\alpha)$.

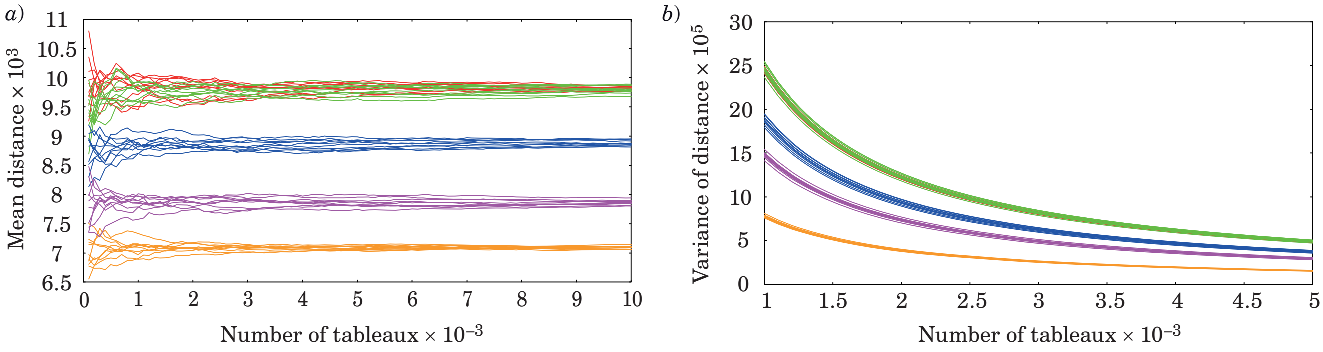
Let us describe how we calculated the coordinates (x^*, y^*) on the limit shape of a bumping route corresponding to the pair of coordinates (x, y) on an experimental path. Firstly, it is necessary to calculate $x_\alpha^{-1}(s)$ from (8). To do this, we find in T_α the value x^* closest to x, which corresponds to some value t. To calculate $y^* = y_\alpha(t)$, t is substituted into (5). To calculate $u_\alpha(t)$ and $v_\alpha(t)$, we find in table T_1 the value of F closest to $\frac{\alpha}{t}$ which corresponds to F^{-1}.

Then, the root-mean-square distance S between the coordinates (x, y) of the boxes of the constructed bumping route and the corresponding coordinates (x^*, y^*) of the limit curve was calculated. Thus, we calculated the distance in the L_2 metric between a stepwise bumping route and the limit shape corresponding to the given insertion value.

To estimate the number k of generated Young tableaux of size $n = 10^6$ required to reliably calculate the standard deviation of distances S for a given value of the input parameter α, a series of random sets of Young tableaux was constructed. For $\alpha \in [0.1, \ldots, 0.85]$, with the step 0.05, the S values of the corresponding bumping route were calculated for each tableau in generated sets. Then, we calculate the averages and variances of S for the first $k = 100, 200, \ldots, 10^4$ tableaux. Figure 2, *a* shows the dependence curves of the average value of S on the number of bumping routes considered. We constructed 10 random sets of 10^4 Young tableaux of size 10^6 for $\alpha = 0.1, 0.3, 0.5, 0.7$ and 0.9.

It could be concluded that the spread in the values of standard deviations decreases significantly with an increase in the number of experiments. Moreover, even if we take 1000 tableaux of size 10^6, the spread can be considered quite acceptable. This conclusion is also confirmed by the variance plot of S values (Fig. 2, *b*) obtained during this experiment — the variance decreases with an increase in the number of tableaux considered, which is in line with our expectations.

Note that Fig. 2, *b* shows only a segment of the graph for the number of tableaux from 1000 to 5000, since the variance curves are difficult to distinguish in a larger scale. For the number of tableaux up to 10^4, the variances still decrease monotonically.



■ ***Fig. 2.*** Dependence of means of S (a) and variances of S (b) on the number of considered Young tableaux of size 10^6 for input values $\alpha = 0.1$ (—), 0.3 (—), 0.5 (—), 0.7 (—), 0.9 (—)

Dependence of the distance between the experimental and limit curves on the size of Young tableaux

In this numerical experiment, the dynamics of convergence of bumping routes to limit curves with increasing size of Young tableaux was studied. The size of considered tableaux varied in the range $n \in [10^5, \ldots, 4\cdot 10^6]$ with the step 10^5. For each size, a fixed number of k pairs of tableaux P and Q were generated: $k = 10^4$ tableaux of sizes from 10^5 to 10^6 and $k = 1000$ tableaux of sizes from $11\cdot 10^5$ to $4\cdot 10^6$. Then, using the RSK algorithm, input values $\alpha \in [0.1, \ldots, 0.85]$ were inserted into each of the obtained tableaux P with the step 0.05.

For each of the k constructed bumping routes of size n, the mean and variance of the root-mean-square distance S were calculated. For a set of fixed values of α, the behaviour of the calculated mean values and variances was studied depending on other parameters.

It is obvious that an experimentally obtained bumping route do not coincide with the corresponding limit shape. Moreover, both their domains of definition and their shapes differ. It was decided to calculate the values of S only on those intervals on which both the experimental bumping routes and their limit shapes were determined. The sections of the bumping routes given by the computer experiment, which go beyond the intervals of existence of the limit shapes, did not participate in the calculation of S.

It was shown in [9] that the deviations of the bumping routes from the limit shapes for large values of n are well approximated by an empirically obtained function of the form

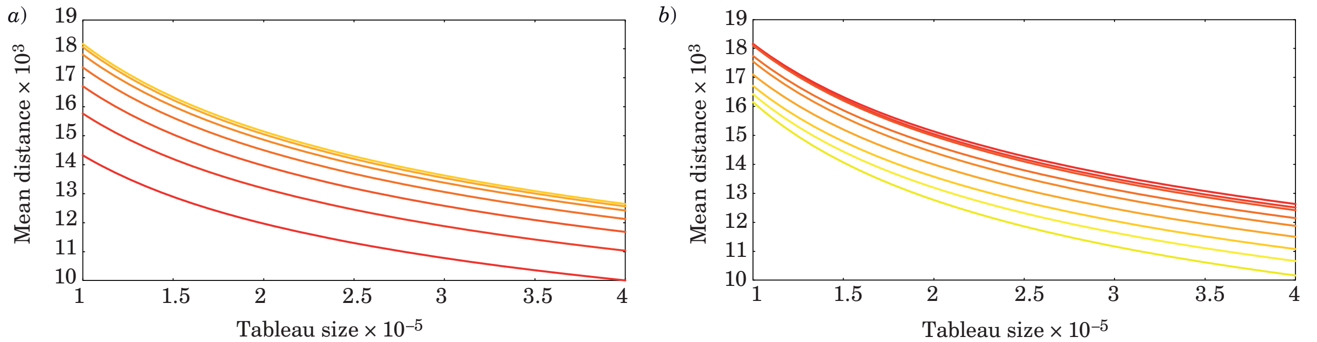
$$f(n) = a\cdot n^{-\frac{1}{4}} + b\cdot n^{-\frac{1}{2}}. \quad (9)$$

Thus, the convergence turns out to be quite slow. The results of calculating the deviations obtained in the framework of computer experiments were approximated using the function (9) with free parameters a and b. The table below shows the values of the obtained parameters for some input numbers α, as well as the maximum deviations Δ of the approximation curves from the experimental bumping routes. The values of Δ show that the approximation accuracy of the empirical formula (9) is quite high.

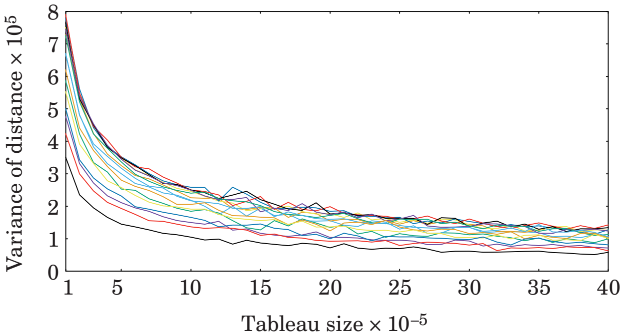
Figure 3, a, b shows approximation curves for input values $\alpha \in [0.1, \ldots, 0.85]$ with the step 0.05. It is easy to see that the largest deviations of the bumping routes from the limit shapes are achieved at $\alpha = 0.4$ and 0.45, and the smallest at 0.1 and 0.85.

■ Values of the coefficients a and b depending on inserted values of α and the maximum deviations Δ of the approximation curves from the experimental bumping routes

α	a	b	Δ
0.1	0.244	0.191	0.000217
0.15	0.270	0.184	0.000239
0.2	0.285	0.213	0.000349
0.25	0.296	0.236	0.000301
0.3	0.302	0.264	0.000209
0.35	0.303	0.325	0.000249
0.4	0.305	0.319	0.000202
0.45	0.304	0.331	0.000298
0.5	0.296	0.475	0.000230
0.55	0.289	0.597	0.000211
0.6	0.281	0.622	0.000233
0.65	0.266	0.826	0.000294
0.7	0.253	0.914	0.000216
0.75	0.233	1.136	0.000270
0.8	0.210	1.457	0.000228
0.85	0.179	1.925	0.000167



■ ***Fig. 3.*** Approximation curves of average distances between bumping routes in Young tableaux with sizes from $n = 10^5$ to $4{\cdot}10^5$ and limit curves corresponding to input values $\alpha \in [0.1(\text{—}), ..., 0.4(\text{—})]$ (*a*); $\alpha \in [0.45(\text{—}), ..., 0.85(\text{—})]$ (*b*) with the step 0.05



■ ***Fig. 4.*** Variances of distances between bumping routes in Young tableaux with sizes from $n = 10^5$ to $4{\cdot}10^6$ and limit curves corresponding to input values $\alpha \in [0.1, ..., 0.85]$ with the step 0.05

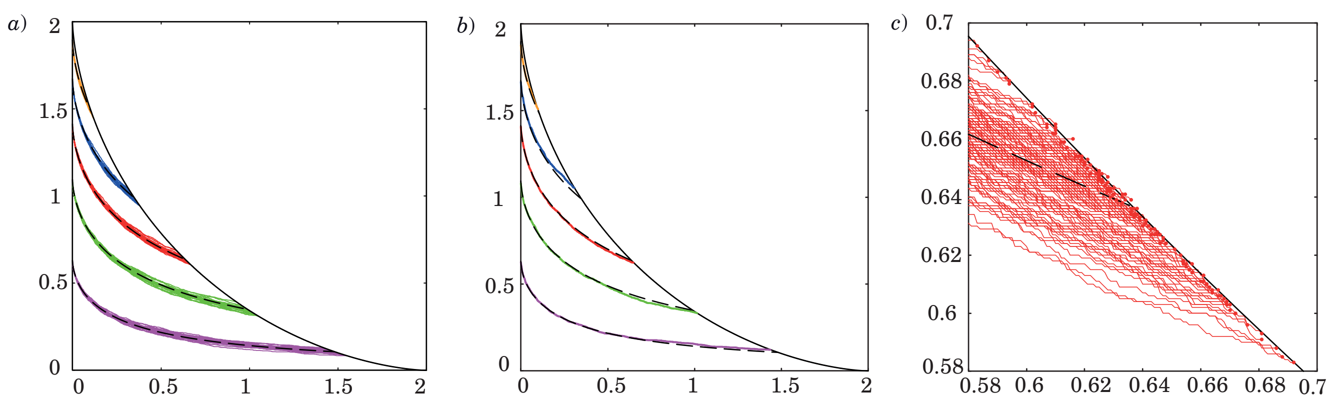
The corresponding variances of the distances between the experimental curves and the limit shapes are shown in Fig. 4. As can be seen, the variances decrease for all considered input values, although not monotonically. Also note that as n increases, the spread of variances for different α decreases.

The coordinates distributions of the ends of the bumping routes obtained in the experiment

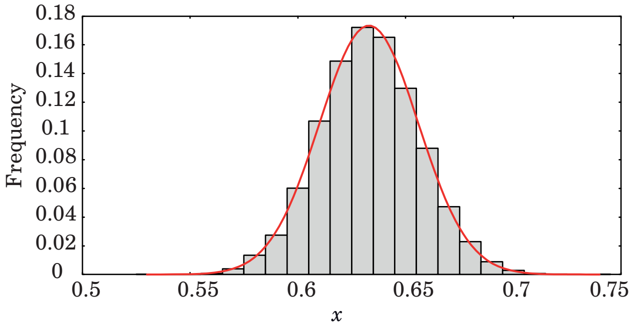
Within the framework of our computer experiments, we have studied the coordinates distributions of the ends of the bumping routes. Also we estimated the values of the parameters of the function approximating these distributions. This experiment actually uses only end points of bumping routes instead of their entire trajectories as the previous one.

In these calculations, we fixed the input value α, the number of considered tableaux k and the size of the tableaux n. Fig. 5, *a* demonstrates 20 bumping routes for each input value $\alpha = 0.1, 0.3, 0.5, 0.7, 0.9$ inserted into a tableau of size 10^6, as well as the Vershik — Kerov limit shape (1). The corresponding limit curves are dashed. Examples of individual bumping routes for the same set of α are shown in Fig. 5, *b*. Figure 5, *c* shows some final parts of the bumping routes in a high zoom level for the case $\alpha = 0.5$. The ends of the bumping routes are marked with red circles.

To study the distribution of the values of the end points, we calculated the number of coordinates belonging to one or another range in x. As a result, we



■ ***Fig. 5.*** The spread of the coordinates of the ends of bumping routes for the input values $\alpha = 0.1$ (—), 0.3 (—), 0.5 (—), 0.7 (—), 0.9 (—), the limit shapes of bumping routes (– –), and the Vershik— Kerov limit shape (—) for 20 (*a*), 1 (*b*) bumping routes, and 100 zoomed final parts of the bumping routes for $\alpha = 0.5$ (*c*)



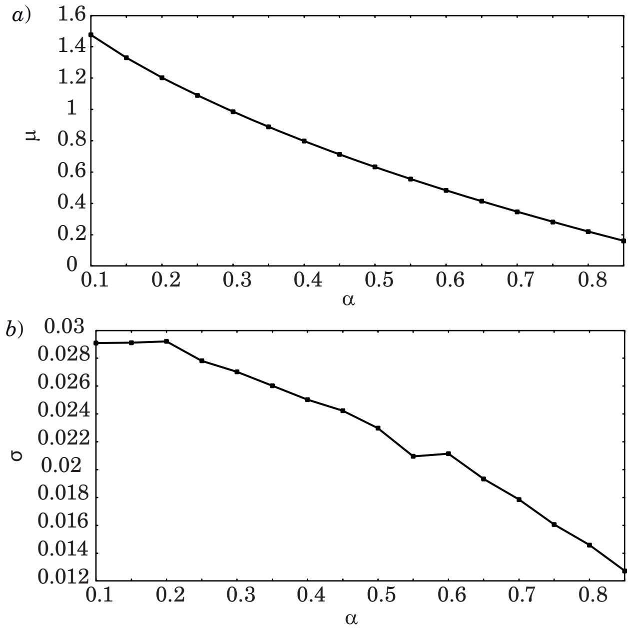
■ ***Fig. 6.*** Frequency histogram of the distribution of the ends of the bumping routes and its approximation using a Gaussian curve

obtained a series of frequency histograms that display how these projections are distributed. An example of one of such histograms is shown in Fig. 6. It is constructed for 10^5 bumping routes in tableaux of size 10^6, when processing an input number $\alpha = 0.5$.

As can be seen from the figure, the distribution of the projection coordinates of the ends of the bumping routes is in good agreement with the normal distribution with the parameters $\mu \approx 0.63$, $\sigma \approx 0.023$. Figure 7, *a*, *b* shows the distribution parameters given by approximating the frequencies of the coordinates of the end points. We considered input values of $\alpha \in [0.1, ..., 0.85]$.

As can be seen from the presented graphs, the dependence of the parameter μ of the normal distribution is close to linear on the studied interval of input values. The parameter σ is estimated less reliably than the amplitude μ, and the results do not allow us to draw a conclusion about the form of such a dependence.

The software package developed to perform the experiments is implemented in C++ with the aid of OpenMP. Experiments use significant computer resources both in terms of memory and time cost. Our calculations were carried out for more than two weeks of continuous computations on the server Ubuntu 20.04.2 LTS, 240Gb RAM, Xeon Gold 5218.



■ ***Fig. 7.*** Dependence of the parameters μ (*a*) and σ (*b*) of the normal distribution for the input values α

Conclusion

Here we have described the results of our computer simulation of the bumping routes in the RSK algorithm obtained on a very large dataset consisting of extra large Young tableaux. We have studied the deviation of these bumping routes from the corresponding theoretical limit shapes.

Our computer experiments show good agreement with the theoretical results obtained in [6] and confirm the empirical asymptotics of the approximation of the bumping routes to their limit shapes. Convergence turns out to be rather slow with the principal term proportional to $n^{-\frac{1}{4}}$. As in many problems of asymptotic combinatorics, such experiments require the involvement of a huge number of large Young tableaux. A similar problem and the RSK algorithm itself, are also defined for the case of strict Young tableaux and the Plancherel process on the Schur graph [15–17]. These two Plancherel processes have a close relationship. The limit shapes of the bumping routes for the strict Plancherel tableaux can be derived from ones in the standard case. But, it should be noted that it is possible to construct much larger Young tableaux on the Schur graph. As a result, it could be possible to obtain much more accurate determination of the parameters of the obtained distributions. So, in the future we plan to extend our research to the strict case. For this, it is also planned to expand the functionality of our specialized software.

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Моделирование путей выталкиваний в алгоритме RSK и анализ их приближения к предельной форме

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Введение: алгоритм RSK устанавливает эквивалентность последовательностей элементов линейно упорядоченных множеств и пар таблиц Юнга P и Q одинаковой формы. Отдельный интерес представляет исследование асимптотического предела, т. е. предельной формы так называемых путей выталкиваний, образуемых выталкиваемыми алгоритмом клетками таблицы P. Точные формулы для этих предельных форм были ранее получены Д. Ромиком и П. Сняды в 2016. Однако проблема изучения динамики стремления путей выталкиваний к их предельным формам остается недостаточно исследованной. **Цель:** исследовать динамику отклонения путей выталкиваний от их предельных форм в таблицах Юнга с помощью компьютерных экспериментов. **Результаты:** в проведенной серии компьютерных экспериментов для таблиц Юнга P размеров до $4\cdot10^6$, заполненных вещественными числами в диапазоне [0, 1], и наборов вставляемых значений $\alpha \in [0.1, 0.15, \dots, 0.85]$ получено большое количество экспериментальных путей выталкиваний, которые сравнивались в метрике L_2 с соответствующими предельными формами. Вычислены средние расстояния и дисперсии их отклонений от предельных кривых. Уточнены параметры в эмпирической формуле для скорости приближения дискретизированных путей выталкиваний к их предельной форме. Получены экспериментальные параметры нормальных распределений отклонения путей выталкиваний для различных входных значений.

Ключевые слова — алгоритм RSK, соответствие RSK, таблица Юнга, мера Планшереля, кривая Вершика — Керова, путь выталкиваний, предельная форма, линейно упорядоченное множество, численный эксперимент, асимптотическая комбинаторика.

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